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BOOK OF ABSTRACTS

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Hybrid "Walk on Equations" Monte Carlo Algorithm for Linear System

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A new hybrid version of Monte Carlo algorithm for solving systems of linear algebraic equations is presented and studied. The algorithm is similar to the "Walk on Equations" Monte Carlo method recently developed by Ivan Dimov, Sylvain Maire and Jean Michel Sellier. For the first time, a comparison with the Gauss-Siedel method for matrices up to 10000 size has been done. The algorithm is improved by choosing the appropriate values for the relaxation parameters which leads to dramatic reduction in time and lower relative errors for a given number of iterations. A theorem for the convergence of the algorithm has been proved. It is shown that the original algorithm can be optimized if we manage to balance the iteration matrix. Also a sequential Monte Carlo method of John Halton based on on an iterative use of the control variate method has been applied. The most important numerical experiment is for a large system coming from a finite element approximation of a problem describing a beam structure in constructive mechanics.

Acknowledgement. Venelin Todorov is supported by BNSF under Project KP-06-N52/5 "Efficient Methods for Modeling, Optimization and Decision Making" and by BNSF under Bilateral Project KP-06-Russia/17 "New Highly Efficient Stochastic Simulation Methods and Applications." Slavi Georgiev is supported by the Bulgarian National Science Fund (BNSF) under Project KP-06-M62/1 "Numerical Deterministic, Stochastic, Machine and Deep Learning Methods with Applications in Computational, Quantitative, Algorithmic Finance, Biomathematics, Ecology and Algebra" from 2022.

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Application of Maximum and Comparison Principles for Systems of Elliptic and Parabolic PDEs with Fully Non-Linear Principle Symbol

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In this talk we present some applications of comparison principle for cooperative systems of fully non-linear elliptic and parabolic equations as existence of viscosity solutions, uniqueness and some quantitative properties of the solution.

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Modern Group Analysis of Ricci Flows

Yu. Bozhkov

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We will present some new results obtained in the joint paper "Symmetries of Ricci Flows" with Enrique Lopez and Stelios Dimas.

To begin with, we will briefly review the Hamilton-Perelman theory based on the Ricci Flow equation, which has culminated in the proof of the celebrated Poincaré Conjecture. Applying the methods of contemporary group analysis of differential equations we find the Lie point symmetries of the Ricci flow on an n-dimensional Riemannian manifold. We propose and apply a method to reutilize these symmetries to obtain the Lie point symmetries of Einstein equations and Ricci Flow equations on warped products of specific manifolds. For the latter, we find invariant solutions.

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Christov Expansion Method for Nonlocal Nonlinear Evolution Equations

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Christov functions allow us to expand derivatives, nonlinear products, and nonlocal (integro-differential) terms back into the same basis. These properties are beneficial when solving nonlinear evolution equations using Galerkin methods. In this work, we demonstrate such a Christov method for the Benjamin–Ono (BO) equation. In the BO equation, the dispersion term is nonlocal, given by a Hilbert transform. To this end, we find the Hilbert transform of the Christov functions using complex integration and Cauchy's residue theorem. Time integration is performed using a Crank–Nicolson-type scheme. Importantly, the Christov method yields a banded matrix for the spatial discretization, even though the spatial terms are nonlocal. To demonstrate the approach and its implementation, we perform numerical experiments showing the overtaking of BO solitary waves.

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Two-Way-Coupled Oscillatory Flows in Compliant Conduits at Arbitrary Womersley Number

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Recently [1], we investigated the fluid–structure interaction (FSI) between the oscillatory flow of a Newtonian fluid and a slender, deformable conduit. Considering the limit of vanishing convective inertia (low Reynolds number) and without imposing constraints on the unsteady oscillation frequency (arbitrary Womersley number), we derived a theory for the instantaneous pressure distribution by two-way coupling the flow with a linearly-elastic deformation law [2]. Specifically, we employed the flow rate–pressure gradient relation from lubrication theory and justified how the deformed tube radius enters the latter. Then, the cross-sectionally-averaged mass conservation equation yields the governing 1D nonlinear PDE for the instantaneous pressure distribution. This PDE is easy to solve numerically. In this talk, I will discuss our recent attempts to obtain perturbative solutions to higher orders in the compliance number, which quantifies how elastic the conduit is relative to the imposed hydrodynamic pressure. At vanishing compliance number, the tube is essentially rigid. For most thin structures (such as Koiter and Donnell shells), the compliance number is expected to be small, suggesting a perturbation expansion. However, this expansion shows certain non-uniform features due to the appearance of a large power of the Womersley number, together with the compliance number, in the pre-factor of the time derivative of the 1D nonlinear PDE. Importantly, this approach allows us to demonstrate that the cycled-average pressure of oscillatory flow in compliant conduits is nonzero, which is interpreted as a new type of FSI-induced "soft streaming" [3] phenomenon.

Acknowledgements. ICC would like to acknowledge the hospitality of the University of Nicosia, Cyprus, where portions of this work were completed thanks to a Fulbright US Scholar award from the US Department of State.

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Oscillating Flow of Rarefied Viscous Gas between Cylinders with Finite Length. Numerical Study

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The rarefied viscous gas flow between a stationary inner and an oscillating outer cylinder is studied in this paper. It is a continuation of our previous publications where we study the flow between the infinite length cylinders while here is considered the case of finite length cylinders bounded by two planes with periodic boundary conditions in the cross sections at the ends of the cylinders. So, on the one hand, this changes the flow character creating conditions for periodic self-organization along the axis of symmetry and on the other hand, the current model is reduced to a flow in a limited volume. A continuum model based on Navier-Stokes-Fourier equations for compressible fluid with first order slip boundary conditions on the cylinders wall and statistical model Direct Simulation Monte Carlo (DSMC) method are used to model the flow. The presented numerical results were obtained after reaching sustained oscillations. Considering the model with cylinders of finite length allows to extend its capabilities and obtain a better fit to real conditions.

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Analytical and Numerical Results on Fluctuation-Induced Casimir and Helmholtz Forces

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The issue of the ensemble's dependence on fluctuation-induced forces pertinent to the ensemble has yet to be studied. Fluctuations are ubiquitous: they unavoidably appear in any matter either due to its quantum nature or due to the nonzero temperature of the material bodies and of the confined medium. When the degrees of freedom can enter and leave the region between the interacting objects one speaks about Casimir force. The now widely investigated critical Casimir force (CCF) results from the fluctuations of an order parameter in the vicinity of a critical point. Recently, a review of the exact results available for the CCF has been published in Ref. [1]. In Letter [2] we have introduced the terms of a Helmholtz fluctuation-induced force. It is a force in which the order parameter value is fixed. We stress, that in customarily considered applications of, say, the equilibrium Ising model to binary alloys or binary liquids, if one insists on full rigor, the case with order parameter fixed must be addressed. We review some recent and present some new both exact and numerical results for the behavior of the Casimir and Helmholtz force

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Development of a Modification of the Original Thermal Comfort Model

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The issue of thermal comfort has been known to man for a long time. In the 1960s/70s, Fanger established the formula that makes it possible to calculate the PMV (Predicted Mean Vote). Since then, some discrepancies have been noticed between this formula and how people actually feel. This indicates that the formula is imperfect and should be revised to better predict people's thermal sensations.

The research was conducted in two ways. The first involved the use of a Testo 400 meter, which analysed and averaged the internal parameters necessary for calculating thermal comfort. The second method aimed to find out the actual thermal sensations of the students during the study by means of questionnaires related to assessment and thermal preferences. The entire execution process of the study was carried out at Kielce University of Technology, in one of the educational buildings. The main objective of the article is to carry out a modification of the Fanger formula with an indicator.

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Mathematical Description of Subjective Symptoms of "Sick Building Syndrome"

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Humans present in buildings can experience various negative symptoms caused by the improper indoor microclimate of the rooms. The present paper is focused on the questionnaire survey conducted at Kielce University of Technology (Poland) of 69 students located in one lecture theatre. The tests took place in the summer conditions. The students expressed their subjective assessment of their well being as well as symptoms of the "sick building syndrome" such as tiredness, sleepiness, dyspnoea. The article presents the frequency of the symptoms' occurrence and their relation to concentration of the students during the lecture. The impact of these symptoms on the students' well – being is also analysed, discussed and generalised in the form of a correlation.

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Numbers of Observations near Order Statistics and Record Values

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Observations near an order statistic are the ones that fall in the left or right vicinity of the given order statistic. Similarly, near-record observations are the ones that occur between successive record times and within a fixed distance of the current record value. These quantities have attracted considerable attention in the literature because they find applications in different fields, including actuarial mathematics, statistical inference, hydrology, meteorology and order-statistics and record theory.

During the talk I will present various properties of random numbers of near-order-statistic and near-record observations and their extensions and generalizations. In particular, I will study their asymptotic properties and indicate practical situations in which these properties are useful. I will not restrict the attention to the case when the observations form a sequence of independent and identically distributed random variables. I will also present some results for the case of strictly stationary sequences of observations.

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Algebraic Geometry and Elliptic Integrals Approach for Calculation of the Propagation Time of a Signal Between GPS Satellites with Account of General Relativity Theory

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The approach for calculating the propagation time of a signal between GPS satellites will be summarized, based on the proposed new theoretical approach in several previous publications, as well as the perspectives for future development of the theory.

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Asymptotic Behavior of the Non-oscillating Solutions of First Order Nonlinear Impulsive Functional-Differential Equations of Neutral Type

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This paper deals with nonlinear first-order impulsive functional-differential equations of neutral type with variable coefficients. The asymptotic behavior of the non-oscillating solutions of such equations is investigated as a function of the values of the variable coefficient in the neutral term to serve as a basis for further studies of the oscillation of the solutions of such equations

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On the 14D Riemann Metric Associated with Navier-Stokes Equations and Its Applications

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The properties of 14-dim Riemann metric, which is a Ricci-flat on solutions of the Navier-Stokes system of equations are studied. With this aim the properties of equations of geodesics of considered metric, written in parametric form are integrated. The examples of flows of viscous incompressible liquids are derived.

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Improving the 3-Junction and 2-Junction Solar Cell Efficiency by Additional Down-Converting UV Light to Visible

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We present the approach to the efficiency improvement of the space 3-Junction and 2-Junction solar cells that involves a layer of down-converting material on the top of the cell. The layer consist of polymer nanocomposite coating impregnated with the nanoparticles of inorganic compounds (fluorides or oxides) doped with rare earth ions, down-converting solar UV light to visible matching the spectrum of the responsivity of the cell [1,2].

We discuss the choice of the polymer host for the nanoparticles to withstand both UV and space radiation and the nanoparticles with maximal down-conversion capabilities. Experimental results demonstrate an increase of the generated current and photoelectric efficiency. A significant improvement of Silicon solar cell efficiency by LUVCON coating, down-converting UV light to near infra-red and therefore increasing the current generated has been observed. UT-Si solar cell coated with LUVCON increased the current by 22.0% compared to cells coated with CORIN polymer only [3]. The mathematical models of 3J and 2J solar cell are presented. 3J solar cells from EMCORE showed a current increase of 5.1% and absolute 2.4% efficiency increase, and 2J solar cell showed current increase by 2.0% and absolute 0.45% efficiency increase, while downconverting coating was not optimized for those 3J and 2J solar cells yet.

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Approximate Analytical Solution for Turbulent Flow in Channels

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Generalized hydrodynamic equations (GHE) proposed by Alexeev (1994) [1], have been successfully used for simulations of incompressible viscous flows for a wide range of problems and flow parameters, including high Reynolds number turbulent flows with thin boundary layers in 3D driven cavity flow at Re = 3200 and 10,000, 2D backward facing step flow at Re = 132,000 Fedoseyev (2012) [2, 3], providing good agreement with experiments Koseff [4] and other works.

The GHE model is not a turbulence model, and no additional equations are introduced. Kinetic effects (small flow scales) are successfully captured with the GHE, and the derived small scale of turbulence compares well with observed in experiments by Koseff and Street (1984). 2D and 3D Navier-Stokes solutions and k- model solutions have been outperformed by GHE results [2]. GHE model has also been applied to compressible supersonic and hypersonic flows that are very challenging problems as such flows can exhibit both continuum and non-continuum flow regimes. The shock wave (bow shock) can be detached from the vehicle at high altitude, and near boundary slip-flow is typical for such regimes. First results for this model has been reported in [5,6,7] are in good agreement with the experiments Allègre (1997) [8], and other works.

In this work we made an attempt to obtain an approximate analytical solution of GHE for flows in channels that accounts for turbulence effects (fluctuations). The analytical solution compared well with experimental data by Van Doorne (2004) [9] and Nikuradse (1933) [10] (Prandtl Lab) for flow in pipes. The analytical solution provides a structure of turbulent boundary layer that coincides with the Wikipedia one [11].

The obtained analytical solution provides a complete structure of turbulent boundary layer that compares well with the experiments by Wei and Willmarth (1989)[12]. The analytical solution presents an explicit verifiable proof that Alexeev generalized hydrodynamic theory (GHE) is in better agreement with experiments for turbulent flows than currently available theories.

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Improving the Space Silicon Solar Cell Efficiency by Down-Converting UV Light to Visible with Adding the Layer of Polymer Nanocomposite

A. Fedoseyev, S. Herasimenka

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It is shown that the efficiency of space silicon solar cell can be improved by applying a layer of down-converting material on the top of the cell. This layer consists of a polymer nanocomposite impregnated with the nanoparticles of inorganic compounds (fluorides or oxides) doped with rare earth ions. It converts the solar UV light to visible matching the responsivity spectrum of the solar cell [1]. The choice of polymer host of the nanoparticles material to withstand both UV and space radiation is discussed, as well as well as and the nanoparticles with optimal conversion efficiency. The experimental results have proved the feasibility of the proposed down converting concept, a significant improvement of Silicon solar cell efficiency by Layer of UV CONverting coating (LUVCON) coating, down-converting UV light to near infra-red and therefore increasing the current generated. Ultra-thin Si (UT-Si) solar cell coated with LUVCON increased the current by 22.0% compared to cells coated with CORIN polymer only. That translates into absolute 4.3% efficiency increase for 22% efficient UT-Si silicon solar cells, resulting in 26.3% efficiency. As UT-Si solar cell degrades very little under space radiation, the EOL efficiency expected to be 26.0%. EOL efficiency of such solar cells becomes the same or higher than the efficiency of ELO-IMM solar cells making even them attractive for the use in space applications [2]. The mathematical model of UT-Si solar cell is presented.

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Radiation Effect Model for Ultrathin Silicon Solar Cells

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We present the experimental and theoretical approach to the model of radiation effects in ultra-thin silicon solar cell subject to electron irradiation. The ultra-thin silicon solar cell produced by Solestial, Inc. intended for use in space to provide power for NASA space missions, DOD, and commercial satellites. The experiments have been done using the electron beam of 1MeV energy, fluences up to 1E15e/cm^2. Theoretical model is based on a Navy Research Lab approach that uses NIEL (Non-Ionizing Energy Loss) as a main parameter. Ultra-thin solar cells do not fit into available models for silicon solar cell radiation effects, therefore new model has been proposed. The experimental setup is described. The comparison of open circuit voltage Voc from the model with the experimental results is presented. Further work includes the models for the maximum power and short current versus the radiation damage, and related experiments.

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New Stochastic Approaches for European Option Pricing

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Lots of challenges in the multidimensional option pricing exist since this is one of the fundamental discipline in large-scale finance problems today. By definition, a European call option provides its holder with the right, but not the obligation, to buy some quantity of a prescribed asset (underlying) S at a prescribed price (strike or exercise price) E at a prescribed time (maturity or expiry date) T. In the contemporary finance the Monte Carlo and quasi-Monte Carlo methods are solid instruments to solve various problems. In the paper the problem of deriving the fair value of multidimensional European style options is considered. Regarding the option pricing problems, Monte Carlo methods are extremely efficient and useful, especially in higher dimensions. In this paper we show simulation optimization methods based on both low discrepancy sequences and variance reduction methods which essentially improve the accuracy of the standard approaches for European style options. The improved accuracy will be crucial for more reliable results for European option pricing. What is more, the suggested approach could be used in situations where the other deterministic methods fail – *e.g.*, in case of high dimensions, complex contract specifications, *etc.*

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Advanced Stochastic Approaches for Fredholm Integral Equations

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Integral equations are of high applicability in different areas of applied mathematics, physics, engineering, geophysics, electricity and magnetism, kinetic theory of gases, quantum mechanics, mathematical economics, and queuing theory. That is why it is reasonable to develop and study efficient and reliable approaches to solve integral equations. For multidimensional problems the existing biased stochastic algorithms based on evaluation of finite number of integrals will suer more from the effect of high dimensionality, because they are based on quadrature points. So we need advanced unbiased algorithms for solving the multidimensional problem which will be developed in this paper. A new unbiased stochastic method for solving multidimensional Fredholm integral equations of second kind was proposed and analysed. We have compared the newly proposed unbiased algorithm with the old unbiased stochastic algorithm for the one dimensional problem and multidimensional problem.

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Convergence of the L_1 Two-Term Equation Scheme

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Fractional derivatives have an important role in modeling processes in biology, physics, chemistry, and engineering. Finite differential schemes are a main approach for numerical solution of models using fractional differential equations. In this paper we investigate the convergence of the numerical solution of two-term ordinary fractional differential equation which uses the L_1 approximation of the fractional derivative. Inequalities for the weights of L_1 approximation are derived and used to prove the convergence of the L_1 scheme for the two-term equation. Conditions for the parameter of the two-term equation and error estimates of L_1 scheme are obtained. Experimental results for the order and error of the L_1 scheme are presented in the paper.

Keywords: fractional derivative, L_1 approximation

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Improving Algorithmic Trading in Cryptocurrencies with ChatGPT-4: Developing a Grid Trading Bot for Optimal Performance in Diverse Market Conditions

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In this research, the aim is to unlock the untapped potential of ChatGPT-4 in revolutionizing algorithmic trading within the realm of cryptocurrencies, with a specific emphasis on grid trading bots. By leveraging the advanced capabilities of ChatGPT-4, this study endeavors to enhance the efficiency and effectiveness of algorithmic trading strategies employed in the volatile cryptocurrency market.

This study sheds light on the transformative potential of ChatGPT-4 in the domain of algorithmic trading for cryptocurrencies, particularly in the context of grid trading bots. Through its ability to identify market structure, implement risk management protocols, and conduct rigorous backtesting simulations, ChatGPT-4 emerges as a promising tool for creating an optimized grid trading bot that leverages the full potential of algorithmic trading in the world of cryptocurrencies. The findings of this study contribute to the advancement of both artificial intelligence and cryptocurrency trading, paving the way for more effective and profitable trading strategies in this dynamic market.

Keywords: algorithmic trading, cryptocurrency, AI

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On Soliton Solutions and Soliton Interactions of Kulish-Sklyanin and Hirota-Ohta Systems

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This report is based on the recent paper [1] and considers a simplest two-dimensional reduction of the remarkable three-dimensional Hirota-Ohta system. The Lax pair of the Hirota-Ohta system [2] was extended to a Lax triad by adding extra third linear equation, whose compatibility conditions with the Lax pair of the Hirota-Ohta imply another remarkable systems: the Kulish-Sklyanin system (KSS) together with its first higher commuting flow, which we can call as vector complex MKdV:

$$\begin{aligned} i\vec{q}_t + \vec{q}_{xx} + 2(\vec{q}^{\,T}.\vec{p})\vec{q} - (\vec{q}^{\,T} s_0 \vec{q})\vec{p}(x,t) = 0, \\ -i\vec{p}_t + \vec{p}_{xx} + 2(\vec{q}^{\,T}.\vec{p})\vec{p} - (\vec{p}^{\,T} s_0 \vec{p})\vec{q}(x,t) = 0, \end{aligned} \tag{1}$$

$$\begin{aligned} \vec{q}_y + \vec{q}_{xxx} + 3(\vec{q}_x^{\,T}.\vec{p})\vec{q} + 3(\vec{q}^{\,T}.\vec{p})\vec{q}_x - 3(\vec{q}^{\,T} s_0 \vec{q}_x) s_0 \vec{p}(x,t) = 0, \\ \vec{p}_y + \vec{p}_{xxx} + 3(\vec{p}_x^{\,T}.\vec{q})\vec{p} + 3(\vec{q}^{\,T}.\vec{p})\vec{p}_x - 3(\vec{p}^{\,T} s_0 \vec{p}_x) s_0 \vec{q}(x,t) = 0. \end{aligned} \tag{2}$$

where $s_0 = \sum_{k=1}^{2n-1} (-1)^{2n-k} E_{k,2n-k}$. Equation (1) with $\vec{p} = \vec{q}^{\,*}$ has been discovered by Kulish and Sklyanin [3] about four decades ago.

Any common particular solution of these two-dimensional integrable systems yields a corresponding particular solution of the three-dimensional Hirota-Ohta system. Using the dressing Zakharov-Shabat method we derive the N-soliton solutions of these systems and analyze their interactions, *i.e.*, derive explicitly the shifts of the relative center-of-mass coordinates and the phases as functions of the discrete eigenvalues of the Lax operator. Next we relate to these nonlinear evolution equations (NLEE) system of Hirota-Ohta type and obtain its N-soliton solutions.

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Extreme Spacings and Expectation Bounds Based on the Decreasing Reversed Failure Rate Distributions

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We consider the sequence of independent and identically distributed random variables that have the decreasing reversed failure rate. For such setting we present the solution to the problem of determining the upper positive mean-variance bounds on the expectations of the extreme spacings, *i.e.*, the differences of the first two and of the last two consecutive order statistics. Spacings have many applications i.a. in the reliability theory, where they correspond to the time elapsed from the failure of one of the reliability system's component to the death of the next one that fails. We also provide the equality conditions for the bounds. The results are obtained with use of the projection method [1-4].

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On the N-Wave Type Equations and Their Gauge Equivalent

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The class of nonlinear evolution equations (NLEE) – gauge equivalent to the N-wave equations related to the simple Lie algebra g are derived and analyzed. The corresponding Lax pairs and the time evolution of the scattering data are found.

The Zakharov-Shabat dressing method is appropriately modified to construct their soliton solutions.

Several examples including ones describing isoparametric hypersurfaces are presented.

The hierarchy of the Hamiltonian structures to the gauge equivalent systems to the N-wave ones is also discussed.

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Speciation and Historical Migration Pattern Interaction: Examples from Pinus Nigra and Pinus Sylvestris Phylogeography

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In this work we propose a scenario of the evolution and speciation of two important forest trees, European Black Pine and Scotch Pine, and their multiple subspecies and varieties. We use macrophylogeographic mtDNA empirical data and the Migrate-n software to evaluate the proposed population model. Molecular clock simulations revealed that INDEL variability in the Pinus mitochondrial genome is relatively old, *i.e.*, from the Pliocene-Miocene epoch, and related to historical tectonic continental fluctuations rather than to climate change at a large geographic scale. For conservation and management biodiversity program recommendations, special attention is given to the relationships between different speciation models, historical migration patterns, and differences between peripheral and central populations. Species evolution involves the mixing of different speciation modes, and every speciation mode has different effects on different DNA types (*e.g.*, mitochondrial vs. chloroplast vs. nuclear DNA). The misbalance between the contributions of different meta-populat-ion census sizes *vs.* effective population sizes to asymmetric migration patterns is the result of different genotypes (and subphylogenetic lines) responding to selection pressure and adaptive evolution. We propose initial minimal size of conservation unit (between 3 and 5 ha) from central and marginal natural area of distribution for both species in the dynamic management system for practical forest genetic diversity management. The proposed physical sizes were determined by the effective population size, effective radius of seed distribution data, forest

density age dynamics, succession pattern, natural selection pressing and species biology.

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Study of the Spread of Covid-19 for the Last Three Years in Bulgaria

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The emergence of Covid-19 at the end of 2019 in Wuhan, China and its three-year "dominion" on a global scale has changed the world dramatically, affecting both industrial development and the economy, education, medicine and other fields of countries around the world. In May 2023, the World Health Organization ended the public health emergency of international concern regarding COVID-19, but still considers a new outbreak of the disease possible [1]. As of the end of May 2023, Covid-19 is a pandemic, but in the future it may acquire an endemic phase. During it, people will continue to get infected and get sick, but in relatively stable numbers [2]. Therefore, based on real data on morbidity it are needed to be able to predict its development in the future (months or years).

The purpose of this article is to study how the real data and predicting data are mapping each other in spread of Covid-19 in Bulgaria. The data of Covid-19 in Bulgaria over the past three years (2020-2022) are used and compared with forecasting data for the spread of the infection in Bulgaria for several months ahead using the statistical package Prophet in the R software. This package has been successfully used for forecasts regarding financial, epidemiological tasks based on time series. The data are available from the Unified Official Portal of Bulgaria, where they are posted every day [3].

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Keywords: real data, COVID-19, time series, statistics, predictions

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Peculiar Properties of Current Function Fields for a spots Chain with Intervals in a Stratified Fluid

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The spots chain dynamics of mixed liquid in a viscous stratified medium is studied numerically. The spots are arranged horizontally at some interval from each other. In the stratified environment of the atmosphere and the ocean, slim horizontal spots of mixed liquid are observed, which appear when internal ocean waves break. A mixed liquid spot has increased salinity and pressure. The spot is rapidly formed and exists for a long time, gradually flattening. The spot horizontal size significantly exceeds its vertical size. Excessive pressure inside the spot causes it to collapse in the form of intrusions into the surrounding oceanic environment. The fields of the stream function for a spots chain, which changes over time at various Reynolds and Froude numbers, is studied. Salinity is chosen as a stratifying component. The problem is described by the Navier-Stokes equations in the Boussinesq approximation. To solve the problem, the SMIF method (Splitting Method for Incompressible Fluid) is used. The finite different scheme of which has such properties as the second order of approximation on spatial variables, minimal scheme dissipation and dispersion. Under the hydrodynamic forces influence, this chain is transformed into a horizontal streak. It is shown that at lower Reynolds and Froude numbers, the stream function field is stabilized earlier. At high Reynolds and Froude numbers, perturbations propagate over the entire computational domain.

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Explainable Machine Learning / Artificial Intelligence (ML/AI) Models

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The research area of eXplainable AI (XAI) is relatively new and is relevant for ML/AI applications in medicine and other fields, where control is transferred to ML/AI systems. The earlier approaches to ML/AI focused on deterministic, rules-based systems mimicking the logic of human experts, which were clear-cut, explainable and interpretable. The more recent approaches to ML/AI train probabilistic and statistical models employing correlations. These newer ML/AI methods have clear advantages, including higher levels of performance, adaptability to more complex inputs (such as images or sound), and scalability, over older rules-based systems. However, they are more complex and less transparent or interpretable. We compare the use of inherently interpretable models with "black box" models that are explained using sophisticated approaches (*e.g.*, Shapley values).

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Building Machine Learning Models to Improve Hypertension Diagnosis

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Hypertension (HT) is a major risk factor for heart disease, stroke, and kidney disease. A diagnosis of HT is based on 2 or more blood pressure (BP) readings taken on 2 or more visits, and such diagnoses may vary depending on the threshold BP values utilized by specific guidelines. To address the task at hand, the 8-chamber framework (8CF) has been recently proposed and validated for HT diagnosis. The 8CF could be used as ground truth in modeling of latent HT diagnosis. We dichotomize the 8CF to define the dependent variable (HT status). Our approach to modeling of latent HT diagnosis with high precision will leverage analytics to "Big Data," such as electronic health records (EHRs). In our presentation, we will review the mathematics underlying the classical Mashine Learning (ML) methods XGBoost (XGB) and Neural Networks (NN), and suggest new extensions. In particular, we combine the XGB and NN to leverage their strengths (which complement each other) into new predictive models. The performance of all algorithms for diagnosing HT will be characterized using the area under the curve (AUC) approach on a big EHR longitudinal dataset. Predictor variable importance and model interpretability will be assessed using the Shapley values approach.

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Spectral Analysis and Long-Time Asymptotics of Complex mKdV Equation

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In this paper, we obtain the long-time asymptotics of the complex mKdV equation via the Defit–Zhou method (nonlinear steepest descent method). The Cauchy problem of the complex mKdV equation is transformed into the corresponding Riemann–Hilbert problem on the basis of the Lax pair and the scattering matrix. After that, Riemann–Hilbert problems are converted through a decomposition of the matrix valued spectral function and factorizations of the jump matrix for Riemann–Hilbert problem. Finally, by solving the last model problem, the long-time asymptotics of complex mKdV equation are derived.

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Spectral Geometry of Transmission Resonances and Applications

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Transmission eigenvalue problems are a type of non-elliptic and non-selfadjoint spectral problems. They are of central importance to wave scattering theory. Recently, several intrinsic geometric patterns of the transmission eigenfunctions were revealed. In this talk, I shall discuss these intriguing discoveries as well as the corresponding applications to invisibility cloaking, super-resolution imaging and artificial mirage.

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In silico Insight into the Multifaceted Role of the Electric Charge in the Activity of the Antimicrobial Peptides

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It is widely acknowledged that the mechanism of action of anti-microbial peptides (AMPs) is based on their cationic and amphiphilic properties, which enable them to interact with negatively charged bacterial surfaces and membranes by causing membrane rupture or interfering with metabolic processes. Although the majority of AMPs are cationic, there are also non-cationic AMPs. In addition, since they are secreted as complex multicomponent substances with antimicrobial activity, the presence of anionic, cationic, and neutral components should be investigated in relation to the biological activity of the substance in question. We present a novel *in silico* based insight into the multifaceted role of the electric charge in the activity of antimicrobial peptides – from governing the formation of the perfect transport system to its versatile involvement in peptide-membrane interactions.

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Temporal-Causal Modeling of Air Pollution in the City of Plovdiv, Bulgaria: A Case Study

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High levels of air pollution are dangerous to human health, which is a current problem for densely populated cities worldwide. Studying this problem can help

detect pollutants' time dependencies on basic meteorological measurements and other factors for future prediction and elaborate corresponding alarms when official upper pollution limits are exceeded. In this work, time-causal models based on previous daily time observations and meteorological measurements in the city of Plovdiv, Bulgaria, are applied. Vector-type temporal-causal models are constructed and analyzed for carbon dioxide (CO2), nitrogen dioxide (NO2), sulfur dioxide (SO2), and fine dust particles below size 10, 2.5, and 1 micron (PM10, PM2.5, and PM1), respectively. Pollution levels are predicted seven days ahead.

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On the Number of Failed Components in a Working Coherent System under Inspections

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We study the number of failed components in a coherent system with possibly dependent and not necessarily identically distributed component lifetimes, see [1]. Based on two different partial information about the status of component failures, obtained either with a single inspection or under double monitoring of the system, we compute the probability of the number of failures in a working coherent system. In the computations we use the concept of minimal paths sets of the system. These results enable us to establish explicit expressions for survival functions of two types of residual lifetimes of a used system.

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Riemann-Hilbert Problem for the Focusing Nonlinear Schrödinger Equation with Multiple High-Order Poles under Nonzero Boundary Conditions

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In this talk, the Riemann-Hilbert problem is developed to study the focusing nonlinear Schrödinger equation with multiple high-order poles under nonzero boundary conditions. Laurent expansion and Taylor series are employed to replace the residues at the simple- and the second-poles. Furthermore, the solution of RH problem is transformed into a closed system of algebraic equations, and the soliton solutions corresponding to the transmission coefficient with an N-order pole are obtained by solving the algebraic system. Then, in a more general case, the transmission coefficient with multiple high-order poles is studied, and the corresponding solutions are obtained in detail. In addition, for high-order pole, the propagation

behavior of the soliton solution corresponding to a third-order pole and the mixed case of a second-order pole and a simple pole are given as example.

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Mixed Poisson Process with Stacy Mixing Variable

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Stacy distribution defined for the first time in 1961 provides a flexible framework for modeling of a wide range of real-life behaviors. It appears under different names in the scientific literature and contains many useful particular cases. Homogeneous Poisson processes are appropriate a priori models for the number of renewals up to a given time $t > 0$. This paper mixes them and considers a Mixed Poisson process with Stacy mixing variable. We call it a Poisson-Stacy process. The resulting counting process is one of the Generalised Negative Binomial processes, and the distribution of its time-intersections are very-well investigated in the scientific literature. Here we define and investigate their joint probability distributions. Then, the corresponding mixed renewal process is investigated and Exp-Stacy and Erlang-Stacy distributions are defined and partially studied.

The paper finishes with a simulation study of these stochastic processes. Some plots of the probability density functions, probability mass functions, mean square regressions and sample paths are drawn together with the corresponding code for the simulations.

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An Exact Solution to the Inhomogeneous Problem for an Elastic Strip with Mixed Boundary Conditions

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This paper presents an exact solution to the inhomogeneous problem of the theory of elasticity for a strip with mixed boundary conditions on its sides. The lower side of the strip is rigidly clamped, while the upper side is free. Inside the strip along the vertical axis, a certain load represented by an integrable function acts. The solution is constructed by using the Papkovich orthogonality relation, which quickly leads to achievement of the goal. It is represented by series in Papkovich–Fadle eigenfunctions, the coefficients of which are determined in a simple explicit form. The final formulas and examples of solutions to boundary value problems are given.

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A System for Bioinformatics Processing of Metagenomic Data

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The rapid advances in the means of information acquisition, storage and processing, has lead to a significant accumulation of large databases in the field of metagenomics. It is a field in biology that deals with the study of genetic material recovered directly from collected environmental or clinical samples. The data collected by metagenomic experiments are huge in size and inherently noisy and may contain fragmented information that can represent more than 10,000 species. Storing, processing and analysing this data requires software products installed on supercomputers.

The authors present the design, development and creation of a computing server system aimed at data processing and analysis in bioinformatics, metagenomics and 16rRNA gene. The specialized software product Galaxy Europe was used for

this purpose. The metagenomics-based method Targeted Amplicon Sequencing was used to perform the analysis. The data is divided into three test collections in a special fasta file format. They are processed in a web-based Galaxy platform and two configuration variants on the high-performance cluster with GPU cards NVIDIA Tesla V100 32 GB. The analysis process is optimized in order to reduce the information processing time. The parallel performance of the three different system configurations for the data used was studied.

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Stochastic modeling of daily air pollution in Burgas, Bulgaria

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Exceeding the norms and limits of atmospheric air pollution causes enormous damage to the population's health and the environment. Determining the factors affecting air quality is a current task in a local, regional, and global scale. In this study, we use daily time series data for the main air pollutants in Burgas, Bulgaria – O3, SO2, NO, NO2, and PM10, to analyze, model, and forecast these levels depending on meteorological factors. For this purpose, the stochastic ARIMA method and ARIMA with transfer functions are applied. Results are obtained for univariate and multivariate time series. Particular attention is paid to the concentrations of the secondary pollutant ground-level ozone (O3), which are modeled as a function of all variables considered. Results were evaluated using root mean square error, mean absolute percentage errors, Theil statistics, and coefficient of determination. Short-term forecasts have been obtained, and model accuracy up to 84% has been established.

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Reconstruction of the Time-Dependent Right-Hand Side in Parabolic Equations on Disjoint Intervals

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We study the inverse problem of reconstructing the time-dependent right-hand side from point observation in a one dimensional parabolic equation on disjoint intervals. This is a challenging and interesting inverse problem, which has many applications in various fields of physics and mechanics. The problem is ill-possed, *i.e.*, very slight errors in the additional input may cause relatively significant errors in the output of the left and right internal right-hand side.

In this work, we construct computational algorithms, using the loaded equation method. First, we perform a decomposition method with respect to the unknown source of the inverse problem solution. Thus it is reduced to a loaded parabolic equation problem. The numerical performance of the approach is realized by finite difference schemes, solved with special Gaussian elimination algorithms. Numerical experiments show the efficiency of the method.

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Numerical Reconstruction of the Mortality Rate of the Forager Bees in a Spatial Model of Pesticide Free

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We use the spatial model of forager bees that incorporate the return to the hive each day [P. Magal, G.F. Webb, Wu, J. Math. Biol. (2020)]. The mathematical model consists of a Cauchy problem for a diffusion-reaction equation with constant diffusion coefficient and space dependent reaction coefficient, which is the mortality rate. The beekeepers are able to measure the density of forage bees on each half hour. This allows us to formulate inverse problem for determination of space-dependent reaction coefficient from final or time-averaging measurements of forager bees density. We introduce quasi-solutions of the problem that minimize nonlinear last-square objective functional. The Frechet gradient of the objective functionals are obtained using a variational method. The numerical conjugate gradient method, based on the sensitivity and adjoint problems is developed. Computer simulations are discussed.

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Mathematical Modeling of Cylindrical Structures under Impulse Loading

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The paper investigates the dynamic behavior of a hollow elastic cylinder having a finite length, placed inside a rigid cylindrical cavity, under a sharp change in internal pressure. A numerical solution to a two-dimensional dynamic problem is developed using the method of spatial characteristics.

The stress-strain state of an elastic cylinder is described by a hyperbolic equations system with two circular conical surfaces as characteristic surfaces. The outer cones correspond to longitudinal waves; the inner ones correspond to transverse waves.

The calculations were performed under various conditions at the ends and outer surface of the cylinder. An analysis of the results for a cylinder with loose ends shows that the absence of gluing of the outer cylinder surface with the shell leads to a significant increase in the velocities of the cylinder points. Under the internal pressure action, the ends move apart, which leads to a significant increase in the radial velocities of the inner channel.

In the case of a rigid fastening of the end surface with a fixed plate, the pattern of the stress-strain state of a cylinder freely inserted into a rigid shell differs little from that of a cylinder rigidly connected to the shell. However, the chosen dependence of the pressure on the coordinate leads to the appearance of zones of "lamination" of the cylinder surface from the cavity.

The results of the study provide data on the behavior of elastic cylinders under various conditions and have structural mechanics and mechanical engineering as potential areas of application. Understanding the dynamic response of the considered structures to the impulse action can improve and optimize cylindrical systems at the design stage.

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Nonexistence of the Weak Solutions to Some Nonlinear Hyperbolic Equations

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Nonexistence of global in time weak solutions of the nonlinear Klein-Gordon and double dispersion equations of fourth and sixth order is investigated. Power type nonlinearity with coefficients depending on the space variable is considered. For initial energy with nonpositive or positive subcritical initial energy necessary and sufficient conditions for nonexistence of global solutions or non-blowing up solutions is proved. For arbitrary positive initial energy sufficient conditions for nonexistence of global solutions are given.

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Application of Neural Networks for Modeling of Biological Data

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The development of fast and reliable methods for predicting the biological activity of substances in computational biology is of a great importance. This

improves the development of new compounds while keeping costs down. An attractive target for docking experiments among many scientists is the Delta-opioid receptor (DOR) and delta-opioid ligands. Their biological efficacy can be measured by various techniques, facilitating the establishment of the relationship between the structure of the compounds and their biological effect. The relationship between the results of computer experiments and the biological activity of these compounds is modeled by using a non-linear neural network. The primary goal of this study is to determine the most appropriate neural network to model the relationship between in vitro and in silico results for DOR and delta-opioid ligands.

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Keywords: neural network, computer modeling, biological efficacy

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Logistic Probability Function: An Analysis and Modification from the Perspective of Reaction Network Theory

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In this work we study some characteristics of growth functions of the logistic type. The standard logistic function and the 2-logistic growth function are solutions of ordinary differential equations derived from the perspective of reaction network theory. These solutions are compared in terms of their shape. We are interested in the new 2-logistic probability distribution and its characteristics. Using the tools of reaction network theory and numerical methods we derive some properties of this distribution.

Keywords: Growth functions of logistic type, Logistic probability distribution, 2-logistic cumulative distribution, Reaction networks

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In Silico Study of the Mechanism of Action of SARS-CoV-2 ORF6

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Although COVID-19 greatly reduced in mortality rates, the decease still poses a serious public health challenge due to various long-lasting negative health effects. The pathogenicity of the SARS-CoV-2 virus is associated with the action of one of its accessory proteins – the Open Reading Frame 6 (ORF6), the most toxic protein of the virus. This small protein plays a crucial role in immune evasion by antagonizing the host cell interferon signaling pathways, thus limiting the immune response to the infection. Blocking its activity is however greatly held up by the lack of experimentally resolved 3D structure of this protein.

Here we report our *in silico* investigation of the structure and mechanism of action of the SARS-CoV-2 ORF6 protein. A 3D structural model of the protein, embedded in a model endoplasmic reticulum membrane was developed using molecular modeling. We demonstrate that ORF6 accomplishes its effects by binding the ribonucleic acid export 1 (RAE1) protein and sequestering it in the cytoplasm. RAE1 binds to multiple ORF6 molecules simultaneously and very stably, which anchors it to the ER membranes in the cytoplasm. These observations were confirmed experimentally by studying the co-localization of both proteins in ORF6 overexpressing and control cells.

Acknowledgements. This research was partially funded by the Bulgarian National Science Fund under Grant KP-06-DK1/5/2021 SARSIMM. Computational resources were provided by the Discoverer supercomputer thanks to Discoverer PetaSC and EuroHPC JU, as well as the BioSim HPC cluster at the Faculty of Physics by St. Kliment Ohridski University of Sofia.

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Mathematical Modeling of Carbon Nanoparticle Distribution in Polymer Composites for High Electrical Conductivity: Implications for Green Technologies

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A mathematical model for the distribution of carbon nanoparticles in a polymer matrix has been developed to achieve high electrical conductivity in polymer composites. The model takes into account factors such as nanoparticle size, shape, concentration, and surface chemistry, as well as polymer matrix properties such as viscosity and surface tension. The model predicts the spatial distribution of nanoparticles within the polymer matrix, which enables researchers to optimize the dispersion process and achieve maximum electrical conduc-tivity. The importance of electrically condu-ctive composites lies in their potential to contribute to the development of green technolo-gies in road transport, solar energy, batteries, and environmental protection. High electri-cal conductivity enables these technologies to operate efficiently, sustainably, and with minimal environmental impact. The mathematical model

provides a valuable tool for designing and optimizing electrically conductive composites for a wide range of applications in green technologies.

Keywords: mathematical model, electrical conductivity, optimizing electrically conductive

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Preparation and Evaluation of Carbon Nanoparticle-Polymer Composites for High Electrical Conductivity in Green Technologies

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The preparation of nano composite materials consisting of carbon nanoparticles (graphene and nanotubes) in a polymer matrix is a promising approach to achieve high electrical conductivity in polymer composites. This study focuses on the preparation of these materials by extrusion of filaments and the use of electrospray to produce solutions of carbon particles for use in the composite. The dispersion of the carbon fillers in the polymer matrix was evaluated using optical microscopy and electron microscopic measurements. The results showed that proper dispersion of the carbon nanoparticles is essential for achieving high electrical conductivity, as agglomerates and clusters can lead to decreased conductivity and reduced mechanical properties. The importance of electrically conductive composites lies in their potential to contribute to the development of green technologies in road transport, solar energy, batteries, and environmental protection. High electrical conductivity enables these technologies to operate efficiently, sustainably, and with minimal environmental impact. This study provides insights into the preparation and evaluation of nano composite materials for high electrical conductivity and highlights their potential applications in various green technologies.

Keywords: graphene, nanotubes, green technologies, composite, filaments

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Assessing the Impact of Speed and Type of Road Surface on the Noise Level in a Passenger Car Cabin

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This paper presents a study of noise pollution in the cabin of a passenger car in the context of various types of road surfaces. The purpose of the study is to understand how the type of road surface can influence the levels of noise inside the car. Sound pressure levels were measured on A-weighted sound pressure levels for 3 types of road surface - crushed stone pavement, asphalt, and cobblestone. The study was conducted at varying speeds of the car within the range of 10-60 km/h. Due to limitations of homogeneous road conditions, a duration of 10 seconds was chosen for each measurement. The sound level meter was placed at a selected measuring point, close to the driver's head. On the data collected in the above-mentioned way, various regression models have been developed to assess the impact of speed and type of road surface on the noise level. It was found that the noise level increases with the roughness of the road surface, and the resulting risks were analyzed. To detail this negative effect, a spectral analysis of the noise was performed using an octave filter built into the sound level meter. In this way, the dynamics of noise depending on the type of road surface were analyzed in each separate frequency band. The spectral analysis is important when choosing measures to limit sound pressure levels in the cabin of a passenger car, as sound-insulating materials have different efficiency depending on the frequency of the noise that needs to be limited. The presented analysis can be used to identify potential health and comfort risks for the driver and passengers.

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Statistic Study of Particulate Matter Air Contamination in the City of Vidin, Bulgaria

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Ambient air quality is vital to the human health. In recent years, the levels of pollutants in the air have been increasingly monitored. Fine particulate matter (PM) present in the air may be of different origins, such as construction sites, unpaved roads, fields, chimneys, volcanoes or fires. PM contamination of the air is one of the most important causes of diseases of the respiratory tract of people.

Most particles are formed in the atmosphere as a result of complex reactions of chemicals such as sulfur dioxide and nitrogen oxides, which are pollutants emitted by power plants, industries and automobiles. In the EU there are many regulations for monitoring the air contamination, for different norms of the air pollutants and aims for improving the air quality. Bulgaria as a member of the European Union has to comply with all these regulations. This paper deals with PM air contamination in the city of Vidin, Bulgaria. Vidin is located in northwestern Bulgaria, on the south bank of the river Danube. On the other side of the Danube is the town of Calafat, Romania. In Vidin during the winter periods of the years there are PM, sulfur and carbon dioxides air pollution greater than accepted norms. Due to the lack of central heating, the population uses mainly solid fuels for heating both in the domestic and in the public sectors. In Vidin there are days with PM10 pollution greater than the daily norm of $50\mu g/m^3$. This paper aims to study PM10 air pollution in the city of Vidin, Bulgaria for the period 2012–2022.

For the study we use official data from the monitoring of air pollution with PM10 in Vidin region, Bulgaria. We apply different statistical methods to study the data and to predict the future pollution with PM10 in the town of Vidin. All results of the study are graphically presented and commented.

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A Policy Iteration Approximation for Stochastic Optimal Control with Random Coefficients

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In this talk, we develop a numerical method to solve an optimal control problem driven by stochastic differential equations (SDEs) with random coefficients. In particular, we solve the associated Hamilton-Jacobi-Bellman equation (HJB) which takes the form of a second order stochastic partial differential equation (SPDE) by deriving a policy iteration method. We also investigate convergence properties of the approximation and its relation to Newton method. We illustrate the usage of the resulting algorithm with example problems from utility maximisation and finance.

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Maximum Product of Spacings Method under Type-I Censoring with Application to a Weibull Step-Stress Model

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This talk considers the maximum product of spacings (MPS) method for finding estimations of the unknown lifetime parameters in simple step-stress accelerated life testing models. The MPS approach is adjusted to address some inference issues under Type-I censoring. Under certain regularity conditions it is proved that the MPS estimator is asymptotically equivalent to the maximum likelihood (ML) estimator. The special case of tampered failure rate model with Weibull lifetimes sharing a common shape parameter on both stress levels is studied in more detail. MPS and ML point estimates are given as solutions of a non-linear system of equations, whereas a modified bootstrap algorithm is suggested for constructing confidence intervals. An extensive simulation study is performed to compare the MPS and ML approaches. Finally, the presented inference procedures are applied to a real life data example.

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Mathematical and Computer Modeling of Age Changes of Basic Anthropometric and Mass-Inertial Characteristics of Bulgarian Females

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Mathematical modeling is one of the modern methods for the determination of the mass-inertial parameters of different segments of the body, as well as of the whole body, and to study the changes of the inertial characteristics during a specific motion of the considered person. The current work presents a 3D mathematical model of the human body, that is also generated in a computer environment. Both the model and its computer generation allow for the calculation of the mass-inertial characteristics of all segments of the body, as well as for the whole human body in specific body positions. We consider such body postures which are presented in the corresponding classification of astronauts' standard positions of interest for

NASA. We study the age changes of the anthropometric as well as of the mass-inertial characteristics of Bulgarian women aged 25-30 versus those aged 30-40. The obtained results are compared with those reported in the literature: by NASA, as well as other authors. The proposed models shall be also helpful in problems appearing in medicine (orthopaedics and traumatology), computer simulations, rehabilitation robotics, and sports, as well as in areas such as ergonomics, simulation of human behavior in space, forensics (body fall, car crash) and more.

Acknowledgements. This work has been accomplished with the financial support by the Grant No BG05M2OP001-1.002-0011-C02 financed by the Science and Education for Smart Growth Operational Program (2014-2020) and co-financed by the European Union through the European structural and Investment funds.

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Statistic Study of Gaseous Air Contamination in the City of Svishtov, Bulgaria

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Polluted air has a negative impact on human health and on the plant and animal life. There are two types of sources of air pollution: human activities such as transport, heating, industry, *etc.* and natural phenomena such as natural fires, volcanic activity, deforestation and others. Substances that are the most common air pollutants are dust particles, carbon and nitrogen dioxides, carbon and nitrogen oxides, hydrocarbons; aldehydes; radioactive substances, heavy metals, sulfur dioxide, ozone, *etc.* Gaseous air pollutants of primary concern in urban environments include sulfur dioxide, nitrogen dioxide, and carbon monoxide. Sulfur dioxide is a colorless gas that is a product of volcanic eruptions and various industrial processes. It is usually released when coal and oil are burned. Nitrogen oxides, especially nitrogen dioxide, are poisonous gases with a pungent, suffocating odor. There are many regulations aimed at the air pollution and pollution monitoring for all European countries. Bulgaria, as a part of the EU, must comply with all these regulations. There are still many places and periods of time in Bulgaria over the years with air pollution is greater than accepted daily norms.

The present paper is dedicated to the study of air pollution with gaseous substances in the area of the city of Svishtov, Bulgaria. The town of Svishtov is located in Northern Bulgaria, on the south bank of the Danube River, where the southernmost point of the river along its entire length is located. A statistical analysis of the level of air pollution in Svishtov based on official data from the monitoring station in the city is presented. The measurements cover the period 2021–2022. All research results are graphically presented and commented in the paper.

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Approaches for Improvement of Reliability of the Prony's Method Computation

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Parametric spectral analysis methods such as the Prony's method can estimate the frequencies and amplitudes of a signal, conforming to their model, with great precision. At the same time, the addition of noise to the signal can lead to a complete model breakdown which leads to erroneous parameter values. This tends to happen for signals with SNR values as high as 20 dB which can be considered unacceptable for real-life applications. The current article explores several possible algorithms which can be applied to already existing parametric spectral analysis methods in order to refine the results and make them more noise resistant. Such algorithms include spectral and singular decomposition-based methods inspired by continuous-time filter design and signal segmentation methods where the results of each segment processing influence the final estimate based on their relative noise levels. An approach based on the use of non-Euclidean norms as a measure of the linear algebraic equation system's solution quality is developed and tested. Initially, the methods are applied to model digital signals, comprised of harmonic components with varying complex frequencies and amplitudes. Additive white Gaussian and impulse noise is added to the model signals. The results are then tested on model data, obtained based on the layered structure theory and simulations, to which the noise is also added. Finally, the methods are applied to the noisy results of a real-life synthetic aperture synthesis results obtained in near and intermediate zones of radiation.

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Raster Image Processing Using 2D Padé-Type Approximations

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We use the two-dimensional Padé-type approximants method which we have developed earlier to reduce the Gibbs phenomenon for the harmonic two-dimensional Fourier series while processing raster images. A definition of a Padé-type functional is proposed. For this purpose, we use the generalized two-dimensional Padé approximation proposed by Chisholm for the case when the range of the frequency values on the integer grid is selected according to the Vavilov method. The proposed scheme makes it possible to determine a set of series coefficients necessary and sufficient for construction of a Padé-type approximation with a given structure of the numerator and denominator. We consider some examples of Padй approximants application to simple discontinuous templates represented as raster images.The study gives us an opportunity to make some conclusions about practical usage of the Padé-type approximation and about its advantages. They demonstrate effective elimination of distortions inherent to Gibbs phenomena for the Padé-type approximant. It is well seen that Padé-type approximant is significantly more visually appropriate than Fourier one. Application of the Padé-type approximation also leads to sufficient decrease of approximants' parameter number without the loss of precision.

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Numerical Modeling and Applications in Using the Computing Power of the Petascale Supercomputer Discoverer

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The environmental modeling (and air pollution modeling in particular) is one of the toughest problems of computational mathematics. All relevant physical and chemical processes in the atmosphere should be taken into account, which are mathematically represented by a complex system of partial differential equations (PDE).

In order to simplify the original PDE system proper splitting procedure is applied. As a result, the initial system is replaced by several simpler submodels, connected with the main physical and chemical processes. Even in case of a local study of the environment in a relatively small area, the model should be calculated in a large spatial domain, because the pollutants can be moved quickly on long distances, driven by the atmosphere dynamics, especially on high altitude. One major source of difficulty is the high intensity of the atmospheric processes, which require a small time-step to be used in order to get a stable numerical solution (at least in the chemistry submodel). All this makes the treatment of large-scale air pollution models a heavy computational task, that requires efficient numerical algorithms. It has always been a serious challenge for the fastest and most powerful supercomputers of their time. Fortunately, Bulgaria is one of the leading countries in Eastern Europe with respect to the supercomputer infrastructure development in the recent years.

After the IBM Blue Gene/P (installed in BSC/NCSA in 2008, now retired) and the Avitohol system (produced by Hewlett-Packard, installed in IICT-BAS in 2015, still operational), Discoverer is the third Bulgarian supercomputer ranked in the TOP500 list of the most powerful supercomputers in the world and the first of the petascale generation. Discoverer's test results exceeded 4.5 petaflops. It was ranked 91st in the list of TOP500 world's most powerful supercomputer systems and 27th most powerful in the European Union by the time of its installation in 2021. The machine is a part of the international project for creation of powerful network of high-performance machines of the EuroHPC Joint Undertaking. The project is co-funded by the budgets of the Bulgarian state and EuroHPC JU.

Keywords: air pollution model, numerical simulations, parallel computing, supercomputer, MPI, speed-up, scalability

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Development of a Three-Phase Solver for VOF Methods into the PHOENICS Code

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This abstract describes a numerical approach to calculate the behavior of three different incompressible fluids separated by interfaces, considering surface tension and viscous effects. The VOF method is used, based on THINC or CICSAM techniques. To model surface tension, the continuum surface force (CSF) model of two-fluid incompressible flow is extended to three-fluid flow. A thermodynamically consistent surface energy is obtained by splitting surface tension between specific phase-related surface tensions. The paper discusses the numerical method used to solve the governing equations of the three-fluid model and presents numerical results demonstrating the accuracy and robustness of the proposed model in simulating interface dynamics and surface tension effects. This work also integrates the approach into the PHOENICS commercial CFD software for use in complex geometries in 2D and 3D. In summary, this paper presents a new numerical approach for studying the behavior of three-fluid systems and shows its potential for practical applications in complex geometries.

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Orthonormal Eigenfunction Expansions for Sixth-Order Boundary Value Problems

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Sixth-order boundary value problems (BVPs) arise in thin-film flows with a surface that has elastic bending resistance [1, 2, 3, 4]. To solve such problems, a complete set of odd and even orthonormal eigenfunctions — resembling trigonometric sines and cosines, as well as the so-called "beam" functions [5, 6, 7] — is derived. These functions intrinsically satisfy boundary conditions (BCs) of relevance to thin-film flows, since they are the solutions of a self-adjoint sixth-order Sturm–Liouville BVP with the same BCs. Next, we propose a Galerkin spectral approach for sixth-order problems; namely the sought function as well as all terms appearing in the differential equation are expanded into infinite series with respect to the eigenfunctions derived earlier. The unknown coefficients in the expansion are determined by solving the algebraic system derived by taking successive inner products with each member of the complete orthonormal (CON) set of eigenfunctions. The proposed method and its convergence are demonstrated by solving a model sixth-order problem.

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Moving Iso-Contour Method for Solving Partial Differential Equations

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The numerical solution of partial differential equations is often performed on a numerical grid, where the grid points are used for estimating the partial derivatives. The grid can be fully static as in Eulerian type of solution method, or the grid points can move during the solution, which is the case in Lagrangian type of method. In the current article, a numerical solution method is presented, where the grid points are located on iso-contours of the two-dimensional field. The method calculates the local movement of the iso-contours according to an evolution equation described by the PDE, and the solution proceeds by moving the grid points towards the calculated direction. Additional stability is obtained by setting the grid points to move along the iso-contour line. To exemplify the application of the method, numerical examples are calculated for the two-dimensional diffusion equation.

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Investigating Emergent Social Network Properties Through Agents Assembly Simulation

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In the past, agent-based models and simulations have been used in psychology-grounded research [1], as well as by cognitive scientists [2]. However, to create comprehensive agent modes, highly dedicated solutions, developed specifically for the needs of a specific research project, were required [2].

In order to ease model development and facilitate cooperation between researchers of different fields, the Agents Assembly (AASM) domain-specific language, and runtime platform, have been proposed [3]. The AASM ecosystem has been previously tested in the development of simple simulations, supporting researchers with very different backgrounds. Among others, the spread of bacterial disease in a hospital building has been jointly developed by researchers with and without medical knowledge [4]. Moreover, experimental validation has shown that Agents Assembly can easily scale to tens of thousands of interacting agents [3]. Experiences gathered during completed projects, experiments, and in-class use of the platform, provided inspiration for further development and improvement of Agents Assembly.

During the presentation, the current version of Agents Assembly will be briefly introduced. Next, its application to the development and implementation of a psychology-grounded model of the spread of fake news will be discussed. As the current research shows [5], the social engagement around shared false news is influencing chances of sharing it further. For observing such cases on large scales, AASM seems to be an ideal choice. During the conference, hands-on training will be available to interested individuals. This may lead to the development of further collaboration with research teams interested in the use of Agents Assembly in their work.

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The Kadomtsev-Petviashvili Hierarchy on Equation Manifolds

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I will present a short summary of the contemporary geometric approach to differential equations due, among others, to Alexander Vinogradov and his diffiety school: I will explain in what sense a differential equation determines, up to some technical conditions, a sub-bundle of a bundle of infinite jets. Then, given an appropriate differential equation E=0, I will show how to construct and integrate a Kadomtsev-Petviashvili hierarchy posed on such a sub-bundle. Finally, I will present a "driven KP hierarchy", depending smoothly on solutions to E=0.

Acknowledgements. This work is fruit of an ongoing collaboration with Jean-Pierre Magnot and Vladimir Roubtsov (both at the Universite d'Angers/CNRS, France).

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On a Numerical Method for Solving a Non-Stationary Problem of Hydrodynamics with an Angular Singularity

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In this paper, we introduced the non-stationary nonlinear Navier-Stokes equations for the flow of an incompressible viscous fluid in rotation form in two-dimensional polygon with an incoming angle on its boundary. To discretize the problem in time, the Runge-Kutta method of the first order is used. At each time step we define an R_ν-generalized solution in the asymmetric variational formulation of the problem. Its existence and uniqueness in weighted sets is established. An estimate related to the conservation of the energy balance of the approximation velocity field is proved. A weighted finite element method is built.

A numerical simulations of several tests based on the proposed method is made. A set of optimal parameters of the method was determined. The convergence rate of the approximate solution to the exact solution is the same for angles more than 180 degrees and significantly higher than those which use classical finite element approaches. The optimal convergence rate is achieved without using a mesh refinement in the vicinity of the singularity point.

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Global Existence and Blow Up of Solutions for Pseudo-Parabolic Equation with Singular Potential

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In this talk, we like to report a study in the initial boundary value problem of pseudo-parabolic equation with singular potential, in order to classify the initial data for the global existence, finite time blowup and longtime decay of the solution. The whole study is conducted by considering three cases according to initial energy: low initial energy case, critical initial energy case and high initial energy case. For the low initial energy case and critical initial energy case the sufficient initial conditions of global existence, long time decay and finite time blowup are given to show a sharp-like condition. Also two different strategies are applied to estimate the upper bounds of the blowup time for the negative initial energy blowup and positive initial energy blowup respectively. And for the high initial energy case, the finite time blowup is proved.

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A Structure-Activity Modeling Relationship among Mu-Opioid Compounds using Machine Learning

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Opiates are among the oldest drugs which are used to treat many medical problems. They are analgesic and sedative drugs that contain opium. The morphine is its most active ingredient and it is a widely used pain reliever despite of its side effects. The main objective of this study is to construct a model which gives the structure-activity relationship among series of mu-opioid ligands and molecular docking results with a model of mu-opioid receptor by using the machine learning. From the relationship that can be found between the docking results and the in vivo test we could predict the biological effect of the newly synthesized ligands.

Keywords: computer modeling, machine learning, biological efficacy

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Soliton Resolution for the Complex Short Pulse Equation with Weighted Sobolev Initial Data in Space-Time Solitonic Regions

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In this work, we employ the $\bar{\partial}$-steepest descent method to investigate the Cauchy problem of the complex short pulse (CSP) equation with initial conditions in weighted Sobolev space $W^{3,1}(\mathbb{R}) \cap H^{2,2}(\mathbb{R})$. Firstly, we successfully derive the Hamiltonian function of the CSP equation based on its Lax pair. Furthermore, the long time asymptotic behavior of the solution $u(x,t)$ is derived in a fixed space-time cone $S(y_1, y_2, v_1, v_2) = (y, t) \in \mathbb{R}^2 : y = y_0 + vt,\ y_0 \in [y_1, y_2], v \in [v_1, v_2]$. On the basis of the resulting asymptotic behavior, we prove the soliton resolution conjecture of the CSP equation which includes the soliton term confirmed by $N(I)$-soliton on discrete spectrum and the $t^{-1/2}$ order term on continuous spectrum with residual error up to $O(t^{-1})$.

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Simple Layer Potential Expansion for Optimization of Contact Interaction Taking into Account Friction and Adhesion

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Friction forces play a crucial role in the formation of contact problems, particularly through adhesion. Taking it into account leads to a two-term friction law, but often, the simplified form of the law is used in studying contact problems, neglecting adhesion. This paper focuses on a quasi-static three-dimensional contact problem involving the movement of a punch along the boundary of an elastic half-space. The investigation considers friction forces, employing a two-term friction law throughout the contact area. The objective is to optimize the pressure distribution beneath the punch.

The shape of the punch serves as the design variable, while the standard deviation of the pressure distribution beneath the punch, originating from a given distribution, is minimized. The optimization problem can be divided into two sequentially solvable sub-problems. The first task involves finding a pressure distribution that minimizes the performance functional, which has a known analytical

solution. The second problem entails searching for the optimal shape of the punch to achieve the previously determined pressure distribution. This problem is solved using the potential of a simple layer, employing an analytical solution method based on the expansion of the potential and the small parameter method. The coefficients characterizing friction and the adhesive component of friction act as small parameters.

One advantage of this method is the ability to obtain closed-form formulas in each approximation, enabling convenient qualitative analysis and practical engineering applications. The calculations and analytical dependence reveal an asymmetric distribution of pressure on the contact area during the movement of an axisymmetric punch.

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Euler Elastic Model & Its Treatment

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Elastic body deformation at different loading conditions has various applications in engineering, soft structures, and soft robotics. It also serves medical purposes, such as comprehending the forms of vesicles and cell membranes and their deformation behavior [1,2,3].

In this talk, we will consider the Euler elastica ring model subjected to uniform external force p under various loading conditions. We will see the solution of Euler elastica equation, curvature-based nonlinear differential equation, through different approximation methods and show that harmonic balance works better compared with the analytical approach (Elliptic Function). Furthermore, we will also discuss the stability diagram of the elastic ring using harmonic balance formulation over the range of p.

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Parallel Scalability of Sparse Solvers for Sparsified Fractional Diffusion FEM Systems

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‘ Anomalous diffusion is a non-local process that describes a wide spectrum of natural processes and phenomena with many applications in science and technology. It is described mathematically by the fractional Laplace operator. In this work we examine the integral definition of the fractional Laplacian modeled with the Riesz potential and discretized with the finite element method. The thus obtained system of linear algebraic equations is dense due to the non-local nature of the fractional Laplacian and is computationally complex to solve. With LU factorization, for example, solving the problem has an $O(n^3)$ computational complexity, where n is the number of unknowns. However, it can be observed that a large amount of the off-diagonal elements have very small absolute values compared to the diagonal elements. Those small off-diagonal elements can be lumped (set to zero and added to their respective diagonal elements) without significant loss of accuracy. In this work we employ a direct sparse parallel solver to the resultant sparse matrix. We analyse the parallel performance and speed-up, as well as the accuracy, varying the fractional power and the lumping threshold.

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Transformations and Learning

M. Stehlík

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Institute of Statistics, Universidad de Valparaíso, Valparaíso, Chile

During the talk we speak on learning mechanisms of data transformation, aggregation, and fusion. We will introduce SPOCU transfer function and provide some of its unique properties for processing of complex data, also with its behavior. Statistical learning will be also discussed.

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Air Pollution Modeling of UNI-DEM Model by using Innovative Stochastic Approaches

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In this paper we develop advanced multidimensional sensitivity analysis, based on some innovative stochastic approaches for performing air pollution modelling on a large-scale model of long-range 2 transport of air pollutants. The Unified Danish Eulerian Model (UNI-DEM) is a very important mathematical model, and can be applied in many different studies related to damaging effects caused by high air pollution levels. We shall use it in this paper to get a reliable answer to a some very important questions regrading environmental protection. We develop some advanced Monte Carlo and quasi-Monte Carlo methods, based on special lattice and digital sequences. In this paper we will improve the digital ecosystem modeling by improving the existence stochastic approaches. The computational efficiency (in terms of relative error and computational time) of the advanced Monte Carlo algorithms for multidimensional numerical integration has been studied to analyze the sensitivity of UNI-DEM model output to variation of input emissions of the anthropogenic pollutants and of rates of several chemical reactions. The algorithms will be applied to compute global Sobol sensitivity measures corresponding to the influence of several input parameters on the concentrations of important air pollutants. The study will be done for the areas of several European cities with different geographical locations.

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Intuitionistic Fuzzy Seasonal Variation Analysis of Marine Biotoxin Distribution in the Black Sea

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It is crucial to establish a seasonal dependence on the levels of marine biotoxins in the Black Sea to impose seasonal prohibitions on the gathering of marine species in this sea and safeguard human health. Fisher's analysis of variance (ANOVA) is a statistical technique for assessing how various variables affect a collection of data. In some cases, research must be carried out in an unclear and fuzzy environment, with missing or insufficient data.

Intuitionistic fuzzy logic, proposed by Atanassov in 1983, is an approach to resolving this imprecision when a data set happens in an uncertain context, is unclear or is absent. Compared to fuzzy data, intuitionistic fuzzy (IF) data also has a hesitation degree.

In 2020, we first suggested the one-way (1-D IFANOVA) and the software utility for its performance, combining traditional variational analysis with modeling opportunities of Index Matrices (IMs) and Intuitionistic Fuzzy Sets (IFSs). The current study focuses on 1-D IFANOVA of the distribution of marine biotoxins in the Black Sea by the "season"factor using 1-D IFANOVA on the IF dataset of the number of marine biotoxins for the period from February 1 to July 31, 2021. We will also compare the findings of 1-D IFANOVA and conventional ANOVA performed on the identical data set.

The 1-D IFANOVA used to determine the seasonal dynamics of biotoxins in the Black Sea provides the paper with its novelty. The suggested IFANOVA can be extended to apply to multidimensional data in both crisp and fuzzy environments.

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On Multi-Fission Interaction of Traveling Waves for the Two-Dimensional Euler Equations

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In the talk, we introduce an approach for modeling traveling waves interactions derived from the Euler Equations (EE) in shallow water and in two space dimensions.

We use the asymptotic expansion method to obtain a class of functions with free parameters that are shown to be of a first order of approximation to EE, the order is comparable to the classical Boussinesq system. We discuss the advantage of the approach in modeling Lab experiments involving long, small amplitude waves with speed close to 1 and multi-fission interactions.

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Finite Difference Scheme for a Higher Order Nonlinear Schrödinger Equation with Periodic Boundary Conditions

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A Finite Difference scheme for a Higher order Non Linear Schrödinger (HNLS) equation will be presented. The equation considers nonlinear terms such as Kerr and Raman effects, and assumes periodic boundary conditions. In this talk I will present the numerical scheme, a preview of some of its properties, and numerical examples of its performance.

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Numerical Determination of Time-Dependent Volatility for Strike Asian Options from Price Measurements

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In this work robust numerical algorithms for numerical identification of time dependent volatility of Asian options by point and integral price observations are developed. The dependent variables are time – t, the underlying risky asset – S and the integral in time underlying asset – I. The Asian options could be reduced to the study of initial boundary problems for ultra-parabolic equations. For solution to the direct problems, two splitting methods are used to transform the whole time-dependent problem of a fixed and floated strike Asian options into two unsteady subproblems of a lower complexity. The positivity property of the numerical methods is established. For solution to the inverse problems, an average linearization in time of diffusion terms of the semi-discrete and fully-discrete initial boundary value problems are used. Then, a decomposition with respect to the volatility of the approximate solution is applied so that the transition to the new time layer is carried out by solving standard parabolic problems, where the evolution variable is I and the space one is S. The correctness of the algorithms is discussed. Numerical experiments, using simulated as well as real data confirm the effectiveness of the present approach.

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Using Replica Exchange Molecular Dynamics for Quantitative Study of Graft Mimics with Varying Topology

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Cyclotides are unique knotted proteins stabilized by three pairs of disulfide bonds; this motif is known as the cyclic cystine knot (CCK). They display a variety of biological activities, including antimicrobial, insecticidal, antitumor, anti-HIV, *etc.* Their conformational stability, temperature resistance, and resistance to proteolytic degradation make them an ideal scaffold for grafting other biologically active epitopes, resulting in the formation of specific conformations and new functions. Experimental evidence suggests that the molecule MCoCP4, derived by grafting a linear derivative of the CP4 Parkinson inhibitor onto loop 6 of the cyclotide MCoTI-I, has a greater chance of being effective in the treatment of Parkinson's disease. To be comprehended, however, are the dynamic properties of such a grafting system. In the current computational investigation, we investigate this system by means of replica-exchange molecular dynamics (REMD) simulations, from which we sketch the free energy landscape. In contrast, long-scale molecular dynamics (MD) simulations were also performed and compared with REMD simulations, demonstrating that REMD simulations are superior in overcoming the energy barrier and generating complete thermally equilibrated conformational ensembles. Several topologies corresponding to the CP4 Parkinson inhibitor grafted on various loops of the cyclotide MCoTI-II were analyzed as well.

We observe substantial differences in the conformational free energy landscapes of the engineered mimics depending on the grafting position, which can provide help in the design of grafted molecules with predefined therapeutic properties.

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BioSim HPC cluster at the Faculty of Physics of St. Kliment Ohridski University of Sofia.

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Application of the $\bar{\partial}$-Dressing Method for Nonlinear Evolution Equations

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The Dbar-dressing method is applied to study the (2+1)-dimensional Jimbo-Miwa equation and the Kundu-Eckhaus equation with nonzero boundary conditions. For the (2+1)-dimensional Jimbo-Miwa equation, based on the characteristic function and Green's function of the Lax representation, the problem has been transformed into a new Dbar problem. A solution is constructed based on solving the Dbar equation with the help of Cauchy-Green formula and choosing the proper spectral transformation. Furthermore, we can obtain the solution formally when the time evolution of the spectral data is determined.

For the Kundu-Eckhaus equation with the nonzero boundary conditions, a Dbar problem with non-canonical normalization condition at infinity is introduced to investigate the soliton solution. The eigenfunct-ion of Dbar problem is meromorphic outside annulus with center 0, which is used to construct the Lax pair. Then the original nonlinear evolution equation and conservation law are obtained by means of choosing a special distribution matrix. The N-soliton solutions of the focusing Kundu-Eckhaus equation with nonzero boundary conditions are discussed on the basis of the symmetries and distribution.

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Furniture Market Demand Forecasting Using Machine Learning

P. Yaneva, H. Kulina

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Researching consumer demand in the market for goods is essential for any business. The purpose of this study is to forecast the demand for furniture of a large manufacturer in Bulgaria. Major factors influencing customer flow, both in company and online stores of the company, are investigated. Daily observations for nearly two years are modeled using Random Forest and CART Ensemble and Bagging. The constructed models describe the sales of furniture with high goodness-of-fit statistics: coefficient of determination up to 93% and determine the order of the factors influencing the demand.

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Ensemble Methods with Machine Learning for Predictive Modeling of Milk Yield for Holstein Cows

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CART regression models may become unstable in prediction due to extreme measurements, improper selection of predictors, or small variation in predictor values. Ensemble methods such as the CART Ensemble and Bagging (CART-EBag) method, which uses multiple trees and the Bagging algorithm, are more suitable for such cases. Over specified models may also be avoided with this method. Aim: To build and test 305-day milk yield models for a sample of data using the CART Ensemble and Bagging (CART-EBag) method with and without principal component extraction.

Methods: The study uses data from lactations of 160 Holstein cows collected from four farms located in different regions of Bulgaria. 12 features of linear type were measured and evaluated. Using the ensemble data mining method CART Ensembles and Bagging, modeling of the rank variable for the 305-day milk yield was carried out depending on 12 external traits and the farm, as well as the extracted latent variables with PCA. All models were cross-validated and their residuals were analyzed.

Results: Two groups of models were built using the CART ensemble method - with PCA applied and without. The models are for groups of 10, 15, 20 and 25 trees. The models were evaluated and compared according to five predictive statistics – RMSE, MSE, MAD, MAPE and R2. The best model are with R2=0.894 and accuracy RMSE=816.01 kg for ensemble of 10 trees.

Conclusions: The CART Ensembles and Bagging method gives better results when combined with the principal components method. It can also be applied if the dependent variable is not distributed close to normal. Can be applied to fully ordinal type predictors. The outlying cases, in particular the highest values of the dependent variable, are not well predicted, which is a common shortcoming of ensemble averaging prediction methods. For models with more than 25 trees, the statistics start to deteriorate and therefore it is not advisable to increase the number of trees.

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Flowpattern in Hydrocyclones: Numerical Simulations with Experimental Verification

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This paper reports a detailed study of the flow in cyclone separators, with the use of most up-to-date computational fluid dynamics simulations, which are validated with positron emission particle tracking (PEPT) experiments tracing the movement of particles through the cyclone. The parameters varied were the viscosity of the carrier liquid, the flowrate and, in the numerical simulations, the inlet configurations of the cyclone, namely one and two inlets and, with the two inlets, a) both at right angles to the cyclone axis and b) angled downwards. The study reveals features of the flow, which have not been seen till now, but are necessary for the understanding and modelling of the separation and purification efficiency of cyclones. The results of the simulations and the close agreement with experiment are a testament to the reliability and accuracy of large eddy simulation (LES), even for flow features as difficult to simulate as the confined strongly swirling flows in cyclone separators. The results show that a contiguous, smooth surface of zero axial velocity does exist and has approximately the shape that has been assumed by modellers. The significant effects of fluid viscosity, underflow and modifications to the inlet are also shown.

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Structure-Preserving Approach for Non-Conservative System

Weipeng Hu

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For a long term, the classical numerical methods, whatever the Euler differential scheme presented by the famous mathematician—Leonhard Euler, or the Runge-Kutta method widely used in the computational mathematics, the sole target is improving the numerical precision of the numerical methods while the geometric properties of the mechanics system described by the mathematical model are ignored. In addition, the differential equations formulating the mechanics systems are continuous, while the numerical methods for which describe the discrete mechanics systems. Based on this viewpoint, the numerical simulation should be performed in the same geometric framework of the mechanics system. Moreover, the properties of the mechanics system should be preserved as much as possible to improve the long-term stability of the numerical solution.

All physical processes can be formulated as the Hamiltonian form with the energy conservation law as well as the symplectic structure if all dissipative effects are ignored. On the one hand, the important status of the Hamiltonian mechanics is emphasized. On the other hand, a higher requirement is proposed for the numerical analysis on the Hamiltonian system, namely, the results of the numerical analysis on the Hamiltonian system should reproduce the geometric properties of which, including the first integral, the symplectic structure as well as the energy conservation law.

Reviewing the definition of the Hamiltonian system, it can be found that the limitation of the Hamiltonian mechanics is the ignoration of the dissipative effects of the mechanics systems, which implies that the analysis methods based on the Hamiltonian systems cannot be used in the real engineering problems with kinds of dissipative effects at all. Thus, we proposed the generalized multi-symplectic framework to resolve this problem, which is the main theoretical contribution of this book. The generalized multi-symplectic framework built a bridge between the geometric mechanics and the engineering problems, which has been illustrated by many application examples presented adequately in this presentation.

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Modeling the Local Failure in a Viscoelastic Material

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A hollow viscoelastic cylinder subjected to a time-dependent impact load is considered. The cylinder is with finite dimensions. The transient load acts upon the internal surface of the cylinder, whereas no loads are applied to outer surfaces. The end surfaces of the cylinder are restrained to move along the direction along the cylinder's axis. The viscoelastic behavior of the material is modeled using linear Boltzmann-Volterra equations. The transient wave process is simulated for various types of hereditary kernels. A local failure criterion is implemented, and the transient behavior of the viscoelastic continuum is investigated for various applied load intensities.

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High Order Integrable Peakon Models and Peakon Solutions in Non-traveling Wave Type

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In this talk, we will present new type of peakon solutions of partial differential equations, called rogue peaked solitons (peakon) provided with an non-traveling wave. Some linear and nonlinear models are taken to illustrate the rogue peakon solutions.

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Inverse Scattering Transform and Soliton Solutions for the Modified Matrix Korteweg-de Vries Equation with Nonzero Boundary Conditions

Zhi-Qiang Li

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The theory of inverse scattering is developed to study the initial-value problem for the modified matrix Korteweg-de Vries (mmKdV) equation with the $2m \times 2m$ ($m \geq 1$) Lax pairs under the nonzero boundary conditions at infinity. In the direct problem, by introducing a suitable uniform transformation we establish the proper complex z-plane in order to discuss the Jost eigenfunctions, scattering matrix and their analyticity and symmetry. Moreover, the asymptotic behavior of the Jost eigenfunctions and scattering matrix needed in the inverse problem are analyzed via Wentzel-Kramers-Brillouin expansion. For inverse problem, the generalized Riemann-Hilbert problem of the mmKdV equation is established by using the analyticity of the modified eigenfunctions and scattering coefficients. The reconstruction formula of potential function with reflection-less case is derived by solving this Riemann-Hilbert problem and using the scattering data. In addition the dynamic behavior of the solutions for the focusing mmKdV equation including one- and two- soliton solutions are presented in detail under the the condition that the potential are scalar and the 2×2 symmetric matrix.

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